Zero-Knowledge Proofs: The Bridge Between Cryptographic Theory and Blockchain Scalability

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whoami

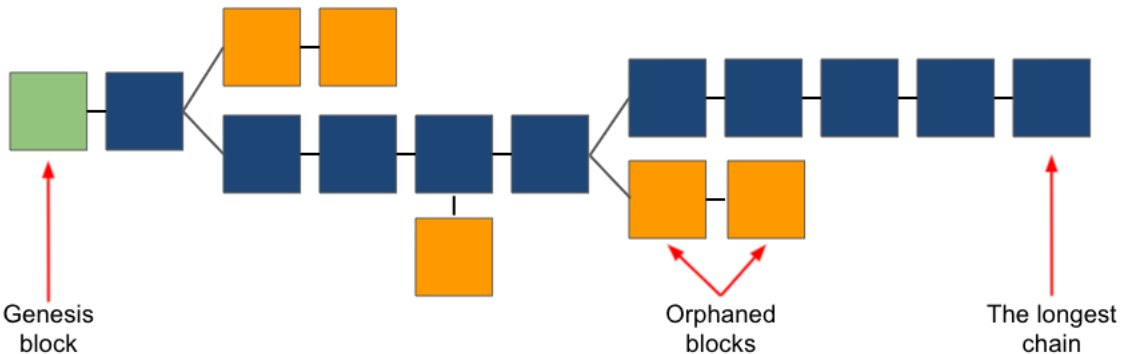
- PhD in cryptography — Ecole Polytechnique (Paris)
- Cryptographer — Consensys (New York)
- Co-developer of gnark and Linea

Overview

1. Blockchains
2. Zero-knowledge proofs
3. Applications
4. Research

Overview

Blockchains



Blockchains

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- How is a tx included in a block?
  - Signatures verification (Bitcoin: ECDSA/Schnorr, Ethereum: ECDSA/BLS)
- How is the longest chain is agreed upon?
  - Consensus (Bitcoin: proof-of-work, Ethereum: proof-of-stake)

Blockchains

A blockchain is a public peer-to-peer *decentralized*, *transparent, immutable, paying* ledger.

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$$\text{Immutable} \xrightarrow[\text{Problem}]{} \text{scalability} \xrightarrow[\text{Solution}]{} ?$$

$$\text{Paying} \xrightarrow[\text{Problem}]{} \text{cost} \xrightarrow[\text{Solution}]{} ?$$

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Zero-knowledge proofs (ZKP)

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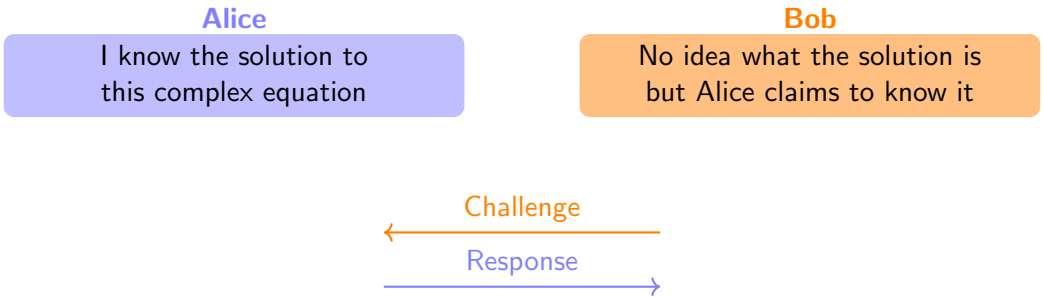

- **Sound**: **Alice** has a wrong solution $\implies$ **Bob** is not convinced.
- **Complete**: **Alice** has the solution $\implies$ **Bob** is convinced.
- **Zero-knowledge**: **Bob** does NOT learn the solution.

ZKP families

Expressivity

- *specific* statement vs. *general* statement

ZKP families

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Deployability

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- *trapdoored* setup vs. *transparent* setup
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- prover complexity (Alice)
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Security

- Cryptographic assumptions
- Cryptographic primitives

Blockchains and ZKP

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setup, prover?, verifier?

Immutable $\xrightarrow[\text{Problem}]{}$ scalability $\xrightarrow[\text{Solution}]{}$ *ZKP*

Communication complexity

Paying $\xrightarrow[\text{Problem}]{}$ cost $\xrightarrow[\text{Solution}]{}$ *ZKP*

Verifier complexity, *prover*?

ZKP literature landmarks

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- SNARK on small fields [plonky, circle-STARK, vortex, ...]

zk-SNARK: Zero-Knowledge Succinct Non-interactive ARgument of Knowledge

"I have a *computationally sound*, *complete*, *zero-knowledge*, **succinct**, **non-interactive** proof that a statement is true and that I know a related secret".

Succinct

A proof is very "short" and "easy" to verify.

Non-interactive

No interaction between the prover and verifier for proof generation and verification (except the proof message).

ARgument of Knowledge

Honest verifier is convinced that a computationally bounded prover knows a secret information.

Preprocessing zk-SNARK for NP language

F: public NP program, x, z: public inputs, w: private input

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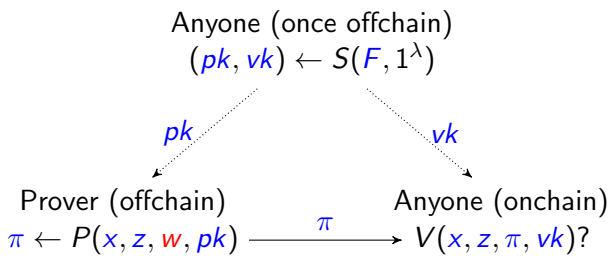

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3. Use homomorphic hiding cryptography to blindly verify the polynomial equation.
4. Make the protocol non-interactive.

A pairing-based SNARK

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- Verify: $0/1 \leftarrow V(\Phi, \pi, vk)$ where V is

$$e(A, B) = vk_{\alpha,\beta} \cdot e(vk_x, vk_\gamma) \cdot e(C, vk_\delta) \qquad (O_\lambda(|\Phi|)) \qquad (1)$$

and $vk_x = \sum_{i=0}^{\ell} [a_i] vk_{\pi_i}$ depends only on the instance Φ and $vk_{\alpha,\beta} = e(vk_\alpha, vk_\beta)$ can be computed in the trusted setup for $(vk_\alpha, vk_\beta) \in \mathbb{G}_1 \times \mathbb{G}_2$.

Overview

Applications

- Privacy: Monero, zcash, Aleo... or Tornado cash...
- Scalability: Mina... or Linea, Aztec...




Monero and Zcash

- Layer-1 blockchains with privacy by design.
- Use ZK proofs to hide transaction data (sender/receiver/amount).

Tornado Cash

- A privacy application on Ethereum (not a base layer).
- Uses ZK proofs to break the link between deposits and withdrawals.

Aleo (ZEXE)

- Uses ZK proofs to hide program executions and state updates.
- ZEXE enables private computations with verifiable correctness.

Mina

- ZK proofs for scalability.
- Fixed-size blockchain via recursive proofs.

Linea and Aztec

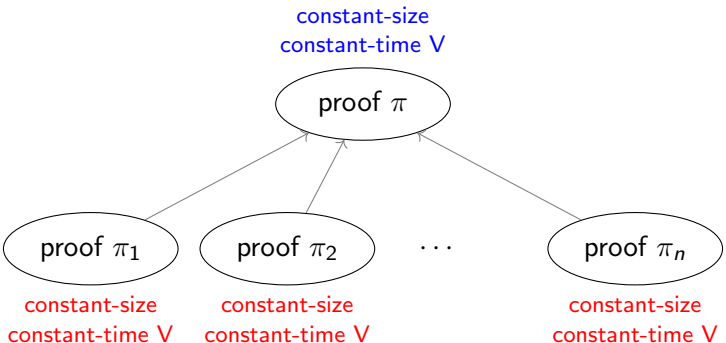
- Layer-2 systems using ZK proofs to scale Ethereum.
- Validity proofs compress many transactions into one.

Ethproofs

- Real-time Ethereum L1 proving.

Overview

Proof composition



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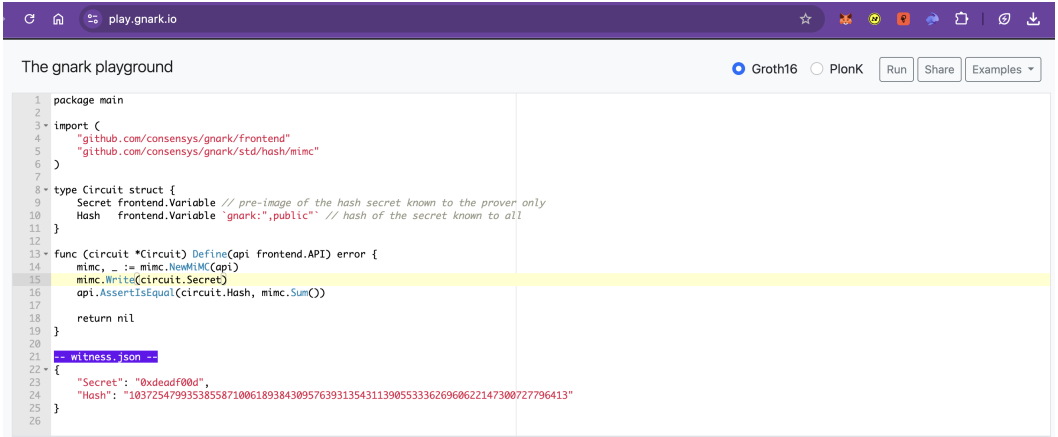
- Implementations: **gnark, linea, arkworks, sonobe, ...**

https://play.gnark.io



Thank you

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- github: `@yelhousni`

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