Modern Applications of Cryptography

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Consensys
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VISA
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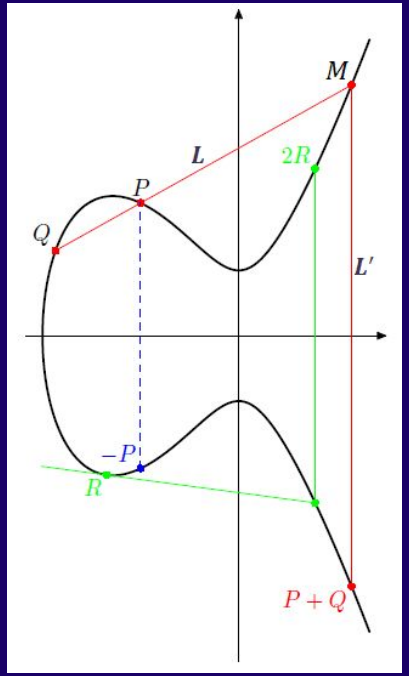
$$f(x) = a_0 + a_1 x + \cdots + x_n x^n, a_i \in \mathbb{Z}/p\mathbb{Z}$$

$$x^{p-1} \equiv 1 \pmod{p}$$



$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$\mathbb{P}(X) \quad \mathbb{P}(X) \quad \mathbb{P}\left(\frac{X}{2}\right)$$

$$\mathbb{P}(X \mid Y) \quad \mathbb{P}(X \mid Y) \quad \mathbb{P}\left(\frac{X}{2} \,\middle|\, Y\right)$$

$$n = p_1^{n_1} \cdot p_2^{n_2} \cdots p_k^{n_k} = \prod_{i=1}^{k} p_i^{n_i}$$

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$$f(x) = a_0 + a_1 x + \cdots + x_n x^n, a_i \in \mathbb{Z}/p\mathbb{Z}$$

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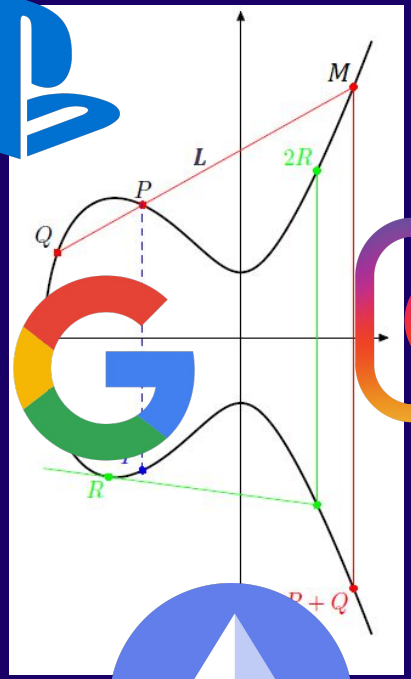

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ & 22 & \cdots & a_{2n} \\ & & \ddots & \vdots \\ & 2 & \cdots & a_{mn} \end{bmatrix}$$

$$n \quad p_2^{n_2} \cdots \quad = \prod_{i=1}$$

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Crypt-o-graphy

- The prefix “crypt-” means hidden
- The suffix “-graphy” means writing

So, all together it says **“hidden writing”**

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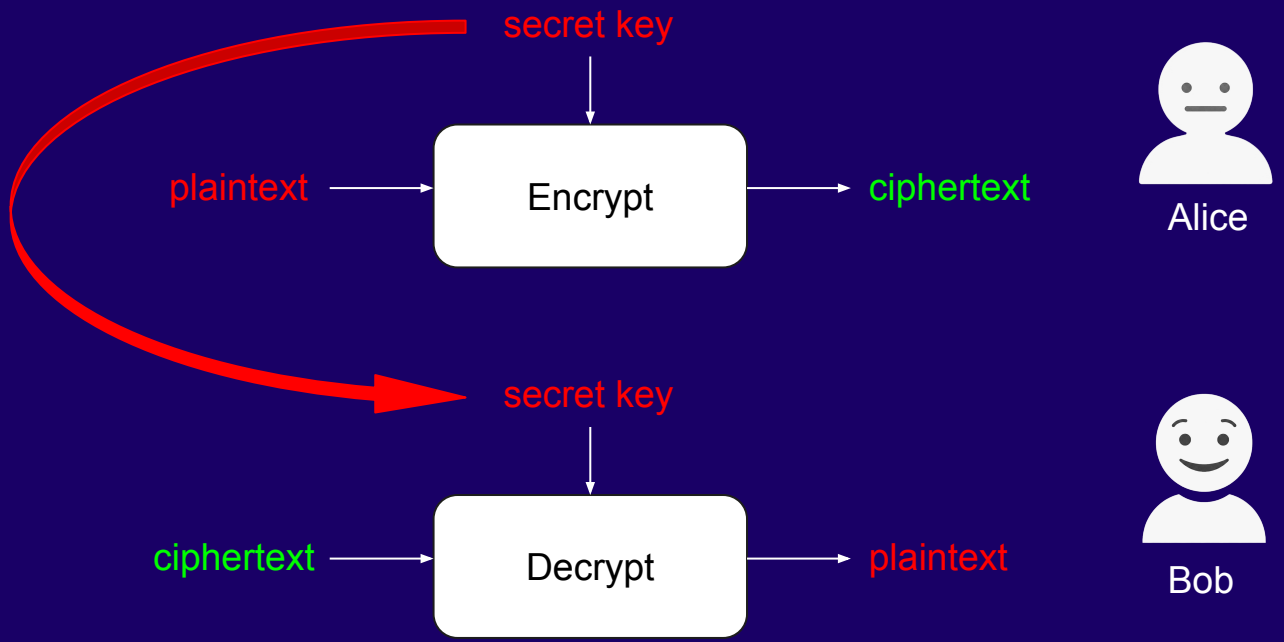
Encryption



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Encryption

Hieroglyphs

Caesar Cipher

Current standard

Enigma WWII

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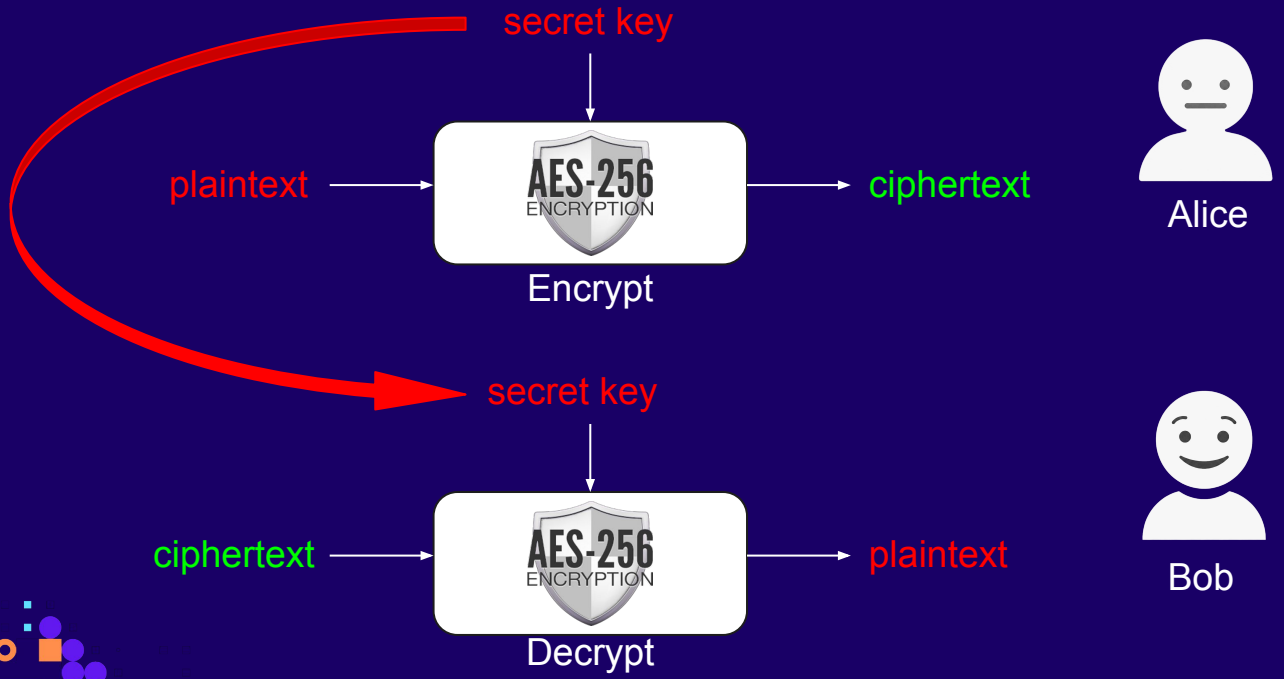
Encryption



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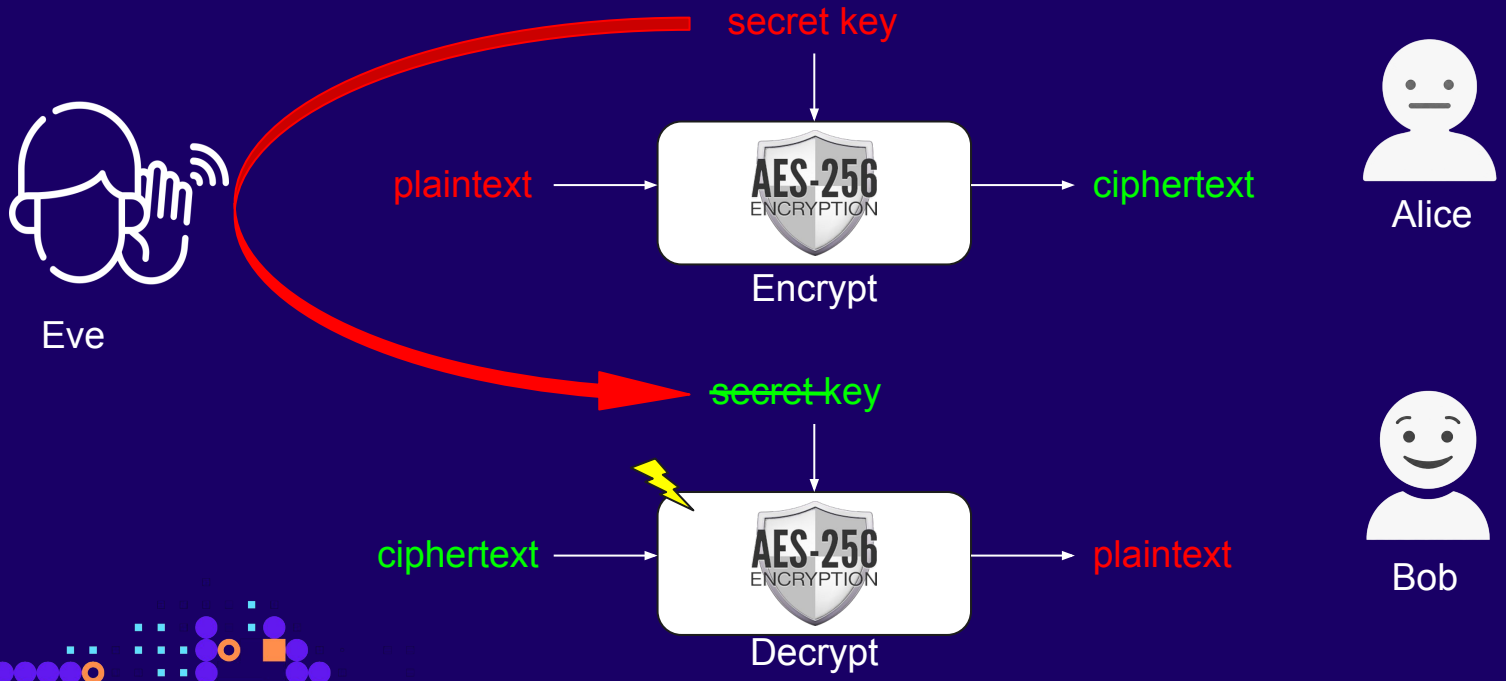
Encryption



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Public key encryption

(secret) key → (public) key

public key

plaintext → AES-256 ENCRYPTION → ciphertext

Encrypt

public key

Alice

secret key

ciphertext → AES-256 ENCRYPTION → plaintext

Decrypt

public key

secret key

Bob

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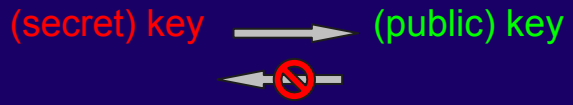

Trapdoor function

$$n = p_1^{n_1} \cdot p_2^{n_2} \cdots p_k^{n_k} = \prod_{i=1}^{k} p_i^{n_i}$$

p_1, p_2 → $n = p_1 \cdot p_2$

$n = p_1 \cdot p_2$ ⊘→ p_1, p_2

6895601 = 1931 x 3571

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Computational complexity

n	2^n	Examples
32	$2^{32} = 10^{9.6}$	number of humans on Earth
47	$2^{47} = 10^{14.2}$	distance Earth - Sun in millimeters ($149.6 \cdot 10^{12}$) number of operations in one day on a processor at 2 GHz
56	$2^{55.8} = 10^{16.8}$	number of operations in one year on a processor at 2 GHz
79	$2^{79} = 10^{23.8}$	Avogadro number: atoms of Carbon 12 in 1 mol
82	$2^{82.3} = 10^{24.8}$	mass of Earth in kilogrammes
100	$2^{100} = 10^{30}$	number of operations in $13.77 \cdot 10^9$ years (age of the universe) on a processor at 2 GHz
155	$2^{155} = 10^{46.7}$	number of molecules of water on Earth
256	$2^{256} = 10^{77.1}$	number of electrons in universe

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RSA Encryption

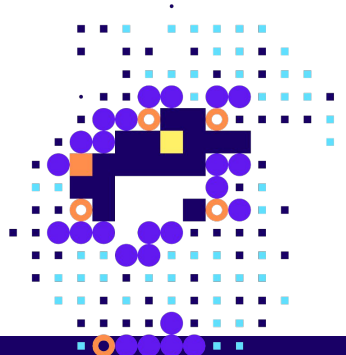
1977, Rivest, Shamir, Adleman

- modulus $N = p \times q$, p, q two distinct large primes
- arithmetic modulo N, in $\mathbb{Z}/N\mathbb{Z} = \{0, 1, \ldots, N-1\}$

The **multiplicative group** is the set of **invertible** integers in $\{1, 2, \ldots, N-1\}$.
invertible x means $gcd(x, N) = 1$, x coprime to N.
There are $\varphi(N) = (p-1)(q-1)$ invertible integers in $\{1, \ldots, N-1\}$

Hard tasks without knowing p, q if N is large enough:

- computing $(p-1)(q-1)$,
- computing a square root $\sqrt{x} = x^{1/2} \bmod N$,
- computing an e-th root $x^{1/e} \bmod N$.



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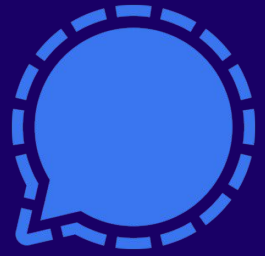
End-to-end encryption

- By default
- Closed-source

- By default
- Closed-source

- By default
- Open-source client
- closed-source server



- By default
- Open-source client
- Open-source server

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(secret) key → (public) key

Signature

$$s \cdot G \to_{\text{easy}} P$$

$$P(= s \cdot G) \to_{\text{hard}} s$$

- Only Bob can sign with s
- Alice (or anyone) can verify with P

Alice

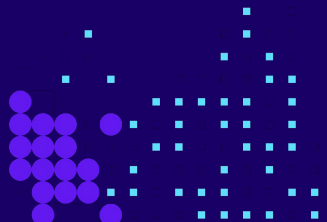
public key P

Bob

public key P

secret key s

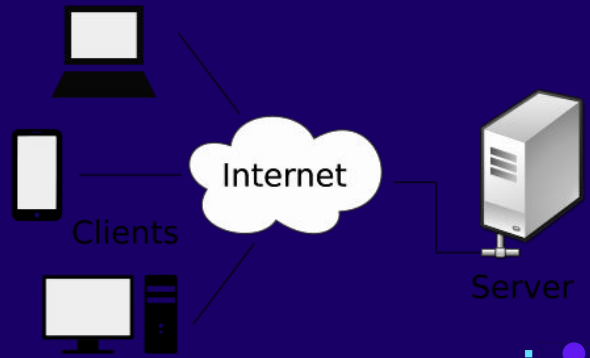
$$x^{p-1} \equiv 1 \pmod{p}$$



Traditional cryptography

https://cipherchallenge.github.io

TLS_ECDHE_RSA_WITH_AES_128_GCM_SHA256, 128 bits, TLS 1.2



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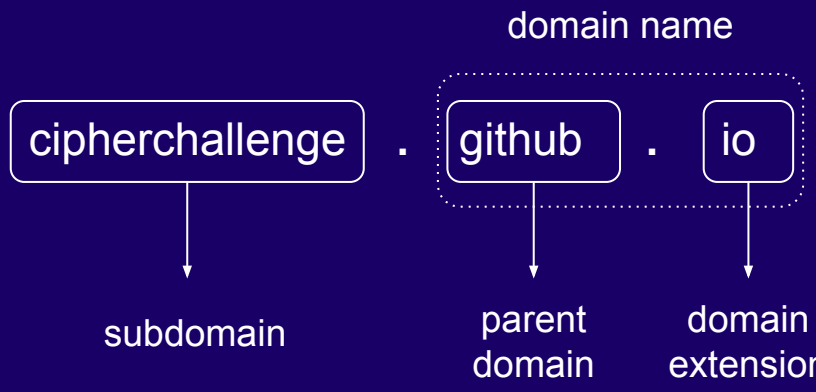
Websites

- Hosting (server)
- Domain name resolution (DNS)



IP addresses for **cipherchallenge.github.io**

Our DNS servers responded with these IP addresses when we queried it for the domain cipherchallenge.github.io. Some DNS servers may return different IP addresses based on your location.

IP address	Type	Hosted by	Location
185.199.108.153	IPv4	Fastly, Inc.	United States of America

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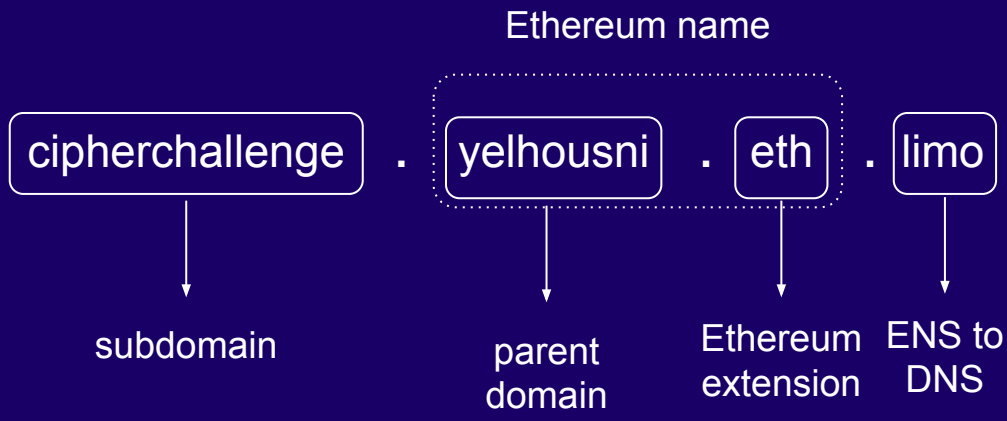
Decentralized Websites

- Decentralized Hosting (p2p IPFS)
- Ethereum name resolution (ENS)



bafybeiclwibybzcxnmbudh5xekmvwhgkhfiod7q4arfimhszru6vhcpjhi.ipfs.dweb.link

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Modern cryptography

$$f(x) = a_0 + a_1 x + \cdots + x_n x^n, a_i \in \mathbb{Z}/p\mathbb{Z}$$

Secret sharing:

Secret key s:
s = s1, s2, s3, …, sn

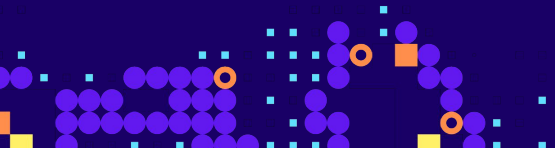
$$s = a_0$$

$$s_i = f(i)$$

E.g. DNSSEC
root key s
s = s1, …, s7
5/7 keys to restart the internet

$$f(1), \ldots, f(n) \rightarrow_{\text{find}} f(x)$$

$$s = f(0)$$



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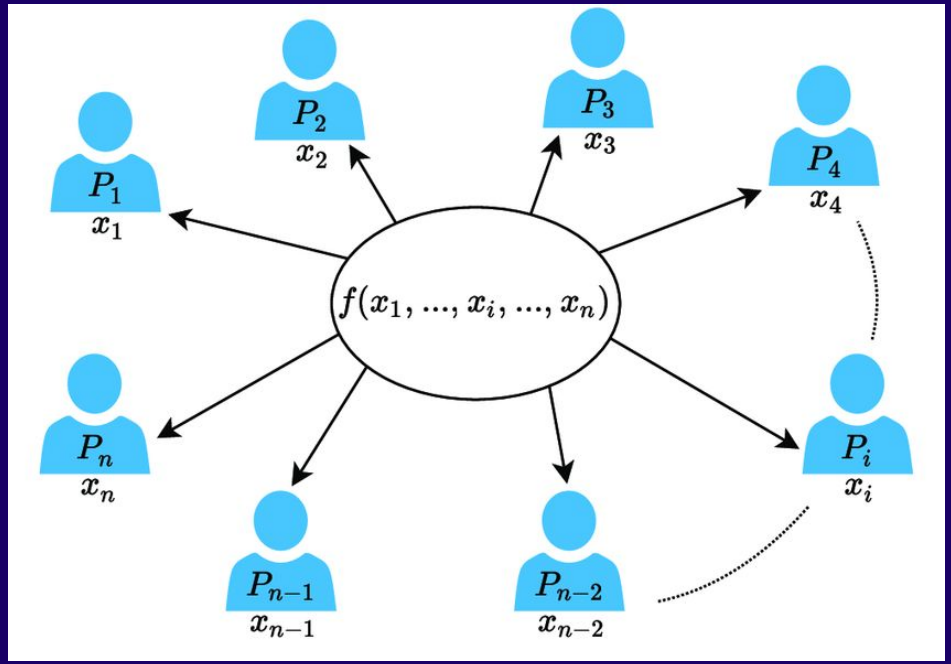
Multi-party computation



N people sign (collaboratively) a transaction

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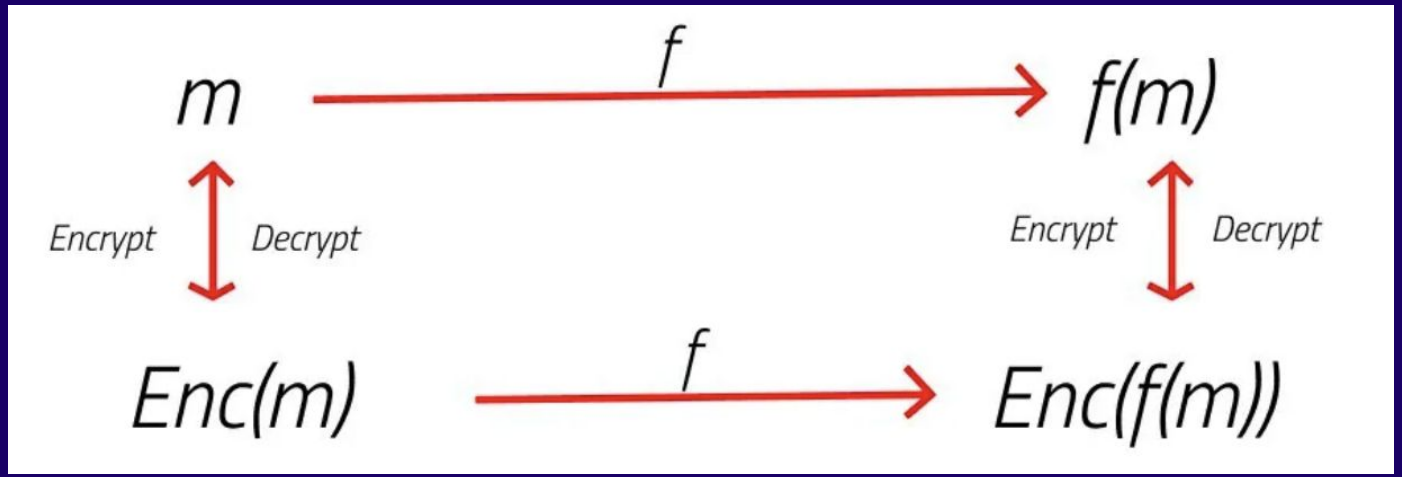
Fully Homomorphic Encryption



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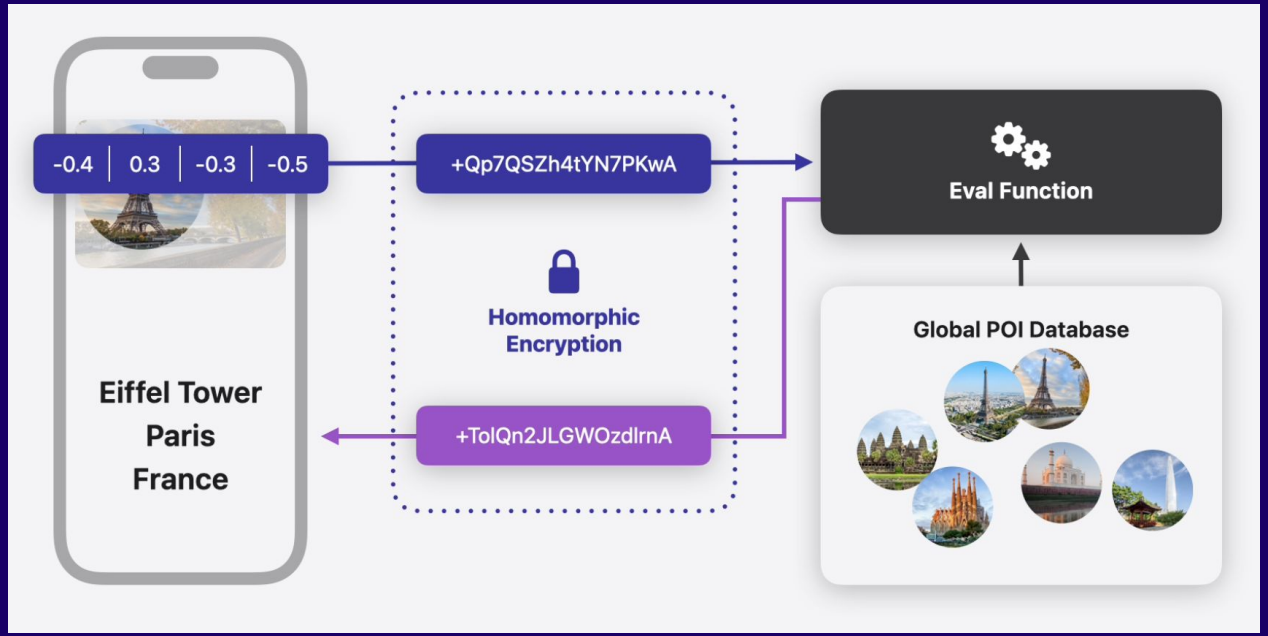
Apple's Private Nearest Neighbor Search



https://machinelearning.apple.com/research/homomorphic-encryption

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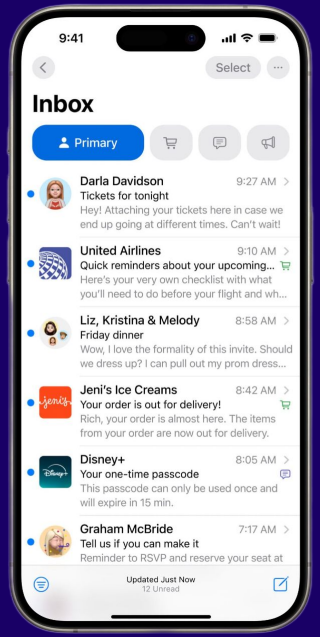
Apple's Private Information Retrieval

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$



https://machinelearning.apple.com/research/homomorphic-encryption

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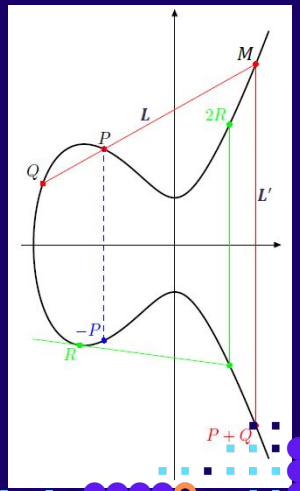
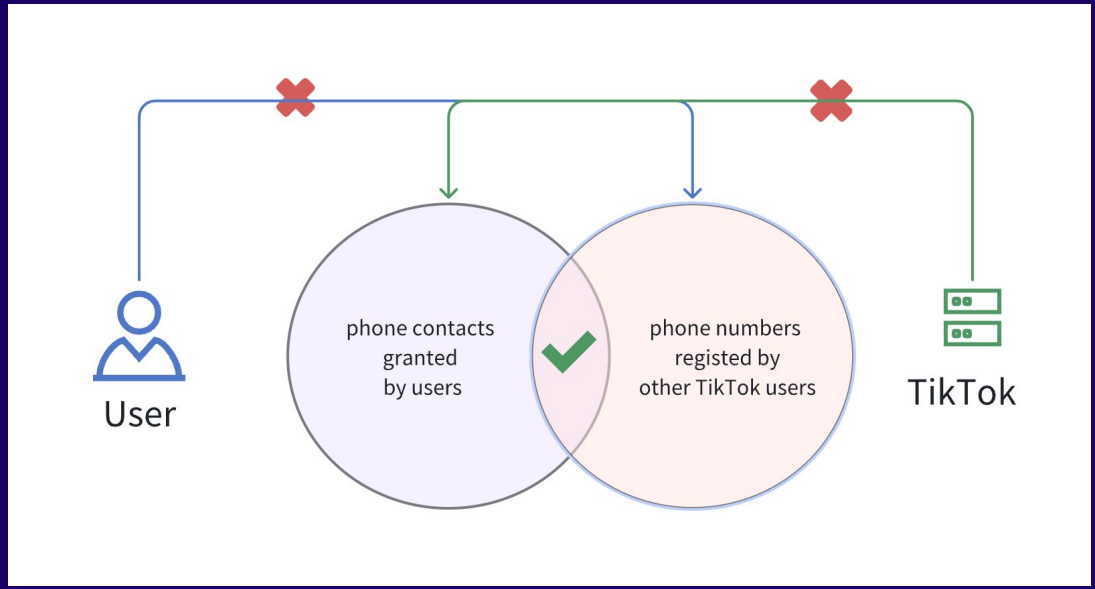
Tiktok's Private Set Intersection





https://developers.tiktok.com/blog/tiktok-practices-in-privacy-enhancing-technologies

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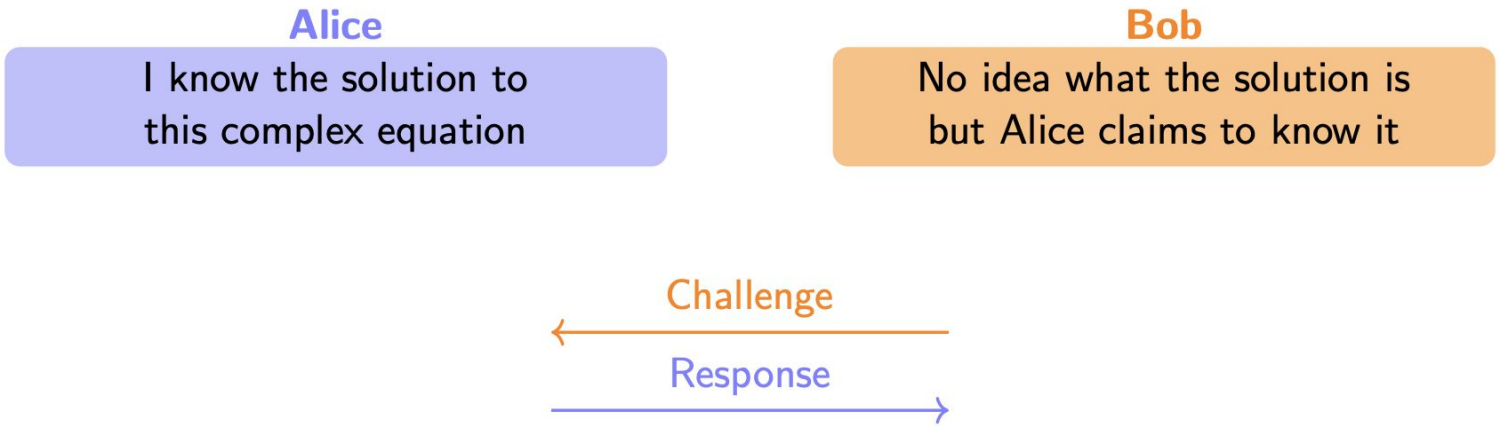
Zero-knowledge proofs



- **Sound**: Alice has a wrong solution $\Longrightarrow$ Bob is not convinced.
- **Complete**: Alice has the solution $\Longrightarrow$ Bob is convinced.
- **Zero-knowledge**: Bob does NOT learn the solution.

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$$f(x) = a_0 + a_1 x + \cdots + x_n x^n, a_i \in \mathbb{Z}/p\mathbb{Z}$$

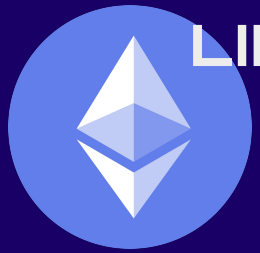
Zero-knowledge proofs

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

I have a zk proof that
X sent $Z to Y

$$\mathbb{P}(X) \quad \mathbb{P}(X) \quad \mathbb{P}\left(\frac{X}{2}\right)$$

$$\mathbb{P}(X \mid Y) \quad \mathbb{P}(X \mid Y) \quad \mathbb{P}\left(\frac{X}{2} \,\middle|\, Y\right)$$



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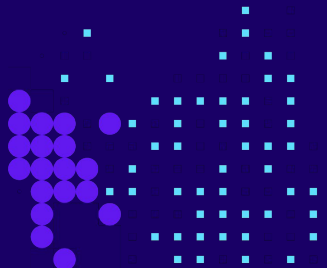
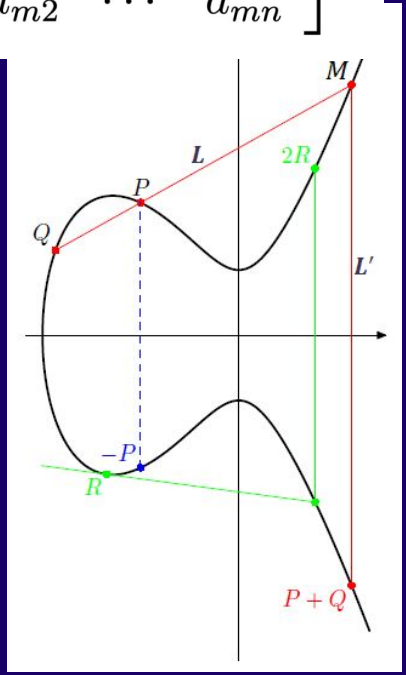
I have a short proof that
the transaction X is correct

|transaction| >> |proof|





https://z.cash
https://www.getmonero.org
https://ethereum.org
https://linea.build

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Zero-knowledge proofs

- Free Cryptography library: https://github.com/Consensys/gnark-crypto
- Free ZKP library: https://github.com/Consensys/gnark
- Playground: https://play.gnark.io/
  - Factorisation example: http://play.gnark.io/?id=petqlbhyng

EdMSM: Multi-Scalar-Multiplication for SNARKs and Faster Montgomery multiplication

Gautam Botrel and Youssef El Housni

Linea, ConsenSys
gautam.botrel@consensys.net

Families of prime-order endomorphism-equipped embedded curves on pairing-friendly curves

Antonio Sanso[1] and Youssef El Housni[2]

[1] Ethereum Foundation
[2] Linea

Pairings in Rank-1 Constraint Systems

Youssef El Housni[1,2,3][0000−0003−2873−3479]

[1] ConsenSys R&D, gnark team, Paris, France
[2] LIX, CNRS, École Polytechnique, Institut Polytechnique d
[3] Inria

Optimized and secure pairing-friendly elliptic curves suitable for one layer proof composition

Youssef El Housni[1,2,3][0000−0003−2873−3479] and Aurore Guillevic[4][0000−0002−0824−7273]

[1] EY Blockchain, Paris, France
[2] LIX, CNRS, École Polytechnique, Institut Polytechnique de Paris
[3] INRIA
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[4] Université de Lorraine, CNRS, Inria, LORIA, Nancy, France
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A survey of elliptic curves for proof systems*

Diego F. Aranha[1][0000−0002−2457−0783], Youssef El Housni[2,3,4][0000−0003−2873−3479], and Aurore Guillevic[1,5][0000−0002−0824−7273]

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[3] LIX, CNRS, École Polytechnique, Institut Polytechnique de Paris
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[5] Université de Lorraine, CNRS, Inria, LORIA, Nancy, France
aurore.guillevic@inria.fr

Co-factor clearing and subgroup membership testing on pairing-friendly curves*

Youssef El Housni[1,2,3][0000−0003−2873−3479], Aurore Guillevic[4,5][0000−0002−0824−7273], and Thomas Piellard[1]

[1] ConsenSys, gnark
youssef.elhousni@consensys.net

Families of SNARK-friendly 2-chains of elliptic curves*

Youssef El Housni[1,2,3][0000−0003−2873−3479] and Aurore Guillevic[4,5][0000−0002−0824−7273]

[1] ConsenSys, gnark, Paris, France
[2] LIX, CNRS, École Polytechnique, Institut Polytechnique de Paris
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aurore.guillevic@inria.fr

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Cryptanalysis
a.k.a. Attacks

- Underlying math
- Implementation
- Side-channel
- Social/frontend
- Political
- Future

An efficient key recovery attack on SIDH

Wouter Castryck[1,2] and Thomas Decru[1]

[1] imec-COSIC, KU Leuven, Belgium
[2] Vakgroep Wiskunde: Algebra en Meetkunde, Universiteit Gent, Belgium

27th Chaos Communication Congress

Console Hacking 2010

PS3 Epic Fail

fail0verflow

bushing, marcan, segher, sven

North Korean hackers cash out hundreds of millions from $1.5bn ByBit hack

10 March 2025

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Apple pulls data protection tool after UK government security row

22 February 2025

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TPM-Fail

TPM meets Timing and Lattice Attacks

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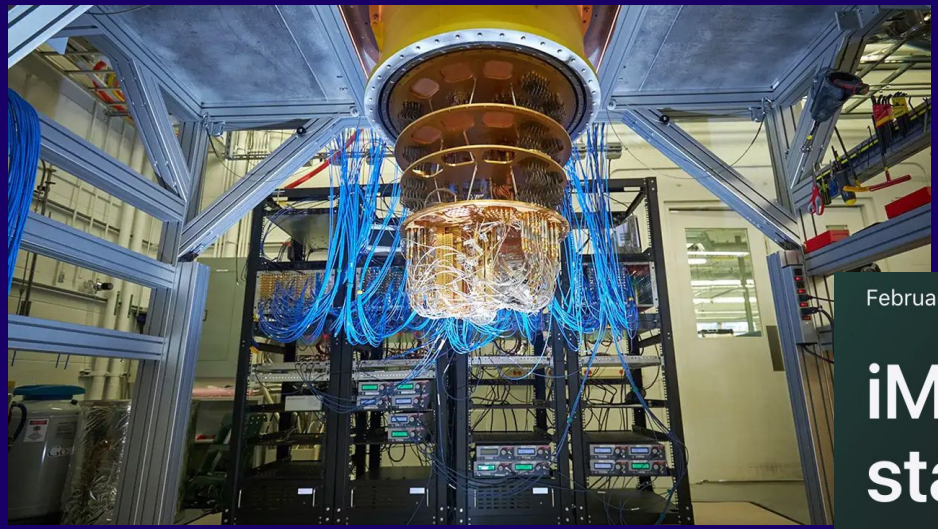
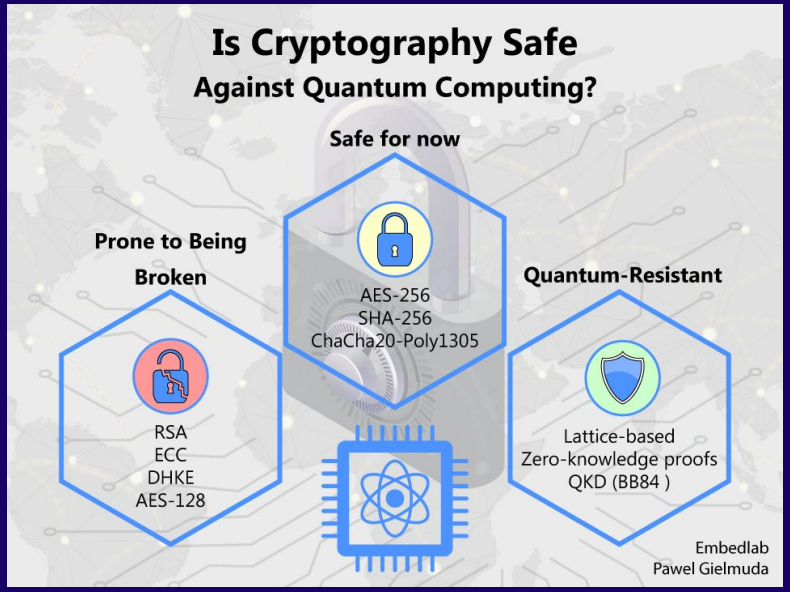
Future attacks



Quantum computer



February 21, 2024

iMessage with PQ3: The new state of the art in quantum-secure messaging at scale

Posted by Apple Security Engineering and Architecture (SEAR)

https://security.apple.com/blog/imessage-pq3/

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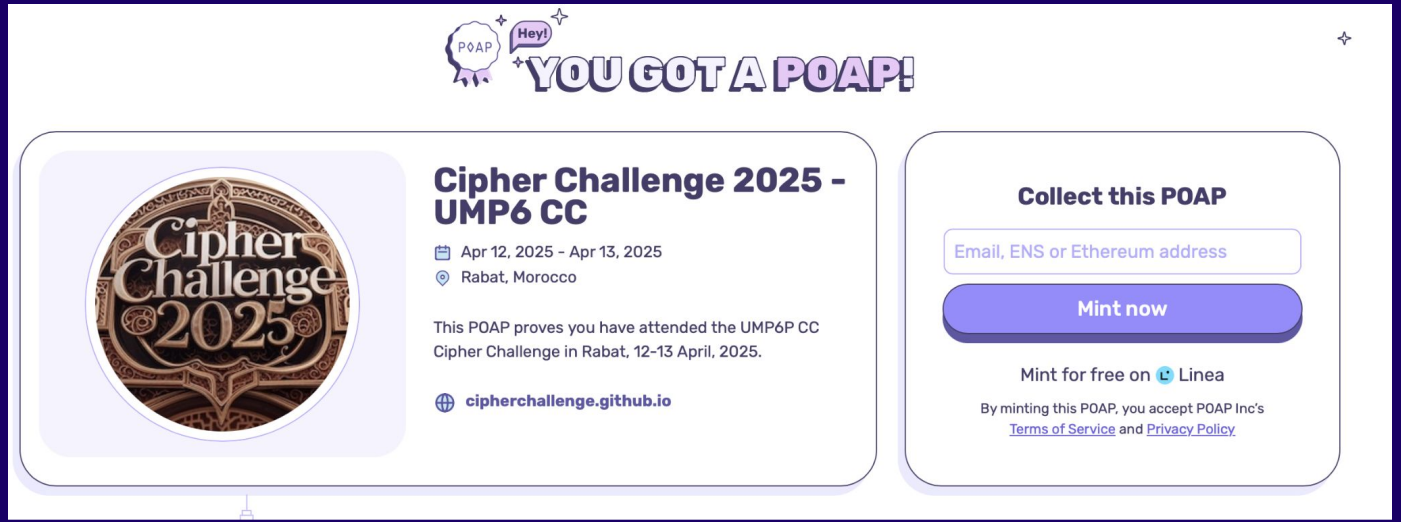
Proof of Attendance Protocol (POAP)



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Thank you

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linea.build
gnark.io

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Telegram: @ElMarroqui
GitHub: @yelhousni
X: @YoussefElHousn3